GE-4 Semester IV

NUMERICAL METHODS

Section I

1. Define Round-off error, Local Truncation error, Global truncation error. If we have three digits arithmetic, add the following numbers from left to right and right to left and compare it using significant digit.
   a) 0.99+0.0044+0.0042
   b) 100+0.49+0.49
2. Give the solution in the interval of chopping and rounding error form
   a) 1.1062 + 0.947 b) 2.747 x 6.823 c) 0.36143447x10^7 – 0.36132346 x 10^7
   d. 0.123 x 10^3 + 0.456 x 10^2
3. Use the standard quadratic formula and rationalized-numerator quadratic formula, rounding to four digits and compare it with true result.
   a) $x^2 - 57x + 1 = 0$
   b) $x^2 - 97x + 1 = 0$
4. Apply Secant Method;
   a. Find the value of √7 in the interval (2,3).
   b. Determine the root lying between (1,2) for the equation $x^3 + x^2 - 3x - 3 = 0$, corrected up to 10^{-4}
5. Determine the smallest positive root of $\cos x - xe^x = 0$ using Regula-Falsi Method and Secant Method.
6. Newton's Method;
   a. Derive the cube root formula for Newton-Raphson method and hence determine $\sqrt[3]{17}\ by\ taking\ x_0 = 2$
   b. Prove that the order of convergence is 2
7. Define the order of convergence of the sequence of iteration. Determine the order of convergence of

   I. $x_{n+1} = \frac{1}{2}x_n\left(3 - \frac{x_n^2}{\alpha}\right)$; $\sqrt{\alpha}\ is\ assumed\ to\ the\ real\ root.$ Ans: 2

   II. $x_{n+1} = x_n\left(\frac{x_n^2+3\alpha}{3x_n^2+\alpha}\right)$; $\sqrt{\alpha}\ is\ assumed\ to\ the\ real\ root.$ Ans: 3

8. Solve the Non- linear system of equation using Newton's Method. (Perform three iteration only)
   I. $f(x,y) = x^2 + y^2 - 1$ take initial iteration $(x_0, y_0) = (0.5,0.5)$
   $g(x,y) = x^2 - y$
   II. $x^2 + xy + y^2 = 7$
   $x^3 + y^3 = 9$ take initial iteration $(x_0, y_0) = (0.5,0.5)$
   III. $x^2 + 4y^2 = 16$
   $xy^2 - 4 = 0$ take initial iteration $(x_0, y_0) = (0.5,0.5)$

Section II

9. Sole the system of equation using Gauss-Elimination Method (with Pivoting).

   I. $4x - 2y + z = 15$
   $-3x - y + 4z = 8$
   $x - y + 3z = 13$

   II. $10x - y + 2z = 4$
   $x + 10y - z = 3$
   $2x + 3y + 20z = 7$

   III. $3x - y + 2z = 7$
   $x + y + 2z = 9$
   $2x - 2y - z = -5$

10. Find the Inverse of the coefficient Matrix of the system using Gauss-Jordan Method.

    I. $x + y + z = 1$
    $4x + 3y - z = 6$
    $3x + 5y + 3z = 4$

    II. A= $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix}$

    III. A= $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

    IV. A= $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 8 \\ 3 & 8 & 14 \end{bmatrix}$

11. Solve using Gauss-Siedel Method (Perform three Iteration) and also stat the condition that Gauss-Siedel converges most rapidly than Gauss-Jacobi Method.

    I. $2x - y = 7$
    $-x + 2y - z = 1$
    $-y + 2z = 1$

    II. $2x - y + z = 1$
    $x + 2y - z = 6$
    $x - y + 2z = -3$

    III. $2x + 3y + z = -1$
    $3x + 2y + 2z = 1$
    $1x + 2y + 2z = 6$

    IV. $2x_1 + 2x_2 = 4$ Solve using Gaussian Tridiagonal method.
    $2x_1 + 4x_2 + 4x_3 = 6$
    $x_2 + 3x_3 + 3x_4 = 7$
    $2x_3 + 5x_4 = 10$

12. Interpolation;

    a. Obtain the Taylor series approximation about x =1, up to second order for the function $f(x) = \frac{1}{1+x^2}$ *over* [1,1.4]. Also find the Truncation error for the approximation. Find the number of terms required in the term to obtain result correct to 5 x 10^{-4}.

    b. Determine the step size h of the function over the interval, so that the truncation error is less than the given value. Also determine the number of terms required to obtain result correct to 5 x 10^{-4}.

       I. $f(x) = \sin x, \left[0, \frac{\pi}{4}\right], \varepsilon = 5 * 10^{-8},$ *for quadratic interpolation.*

II. $f(x) = xe^x,\ [1,2], \varepsilon = 1 * 10^{-5},\ for\ Linear\ interpolation.$

III. $f(x) = (1-x)^{\frac{1}{2}}, [0,1], \varepsilon = 5 * 10^{-3},\ for\ Linear\ interpolation.$
Also find the number of terms required.

c. Let n≥0, let f(x) have n+1 continuous derivatives on [a,b], and let $x_0, x_1, x_2, \ldots\ldots x_n$ be distinct node points in [a,b]. Then,

$$f(x) - p_n(x) = \frac{(x-x_0)(x-x_0)\ldots\ldots\ldots\ldots(x-x_0)}{(n+1)!} f^{n+1}(\xi_n)$$

For $a \le x \le b, where\ \xi_n\ is\ an\ unknown\ point\ between\ the\ minimum\ and\ maximum\ of$ $x_0, x_1, x_2, x_3, \ldots. x_n,$

d. Given that f(0)=1, f(1) = 3 , f(3) =55. Fine the unique interpolating polynomial of degree <2, which fit the given data. Find the bound on the error.

e. Let f(x) = 1/x, then prove that;

$$f[x_0, x_1, x_2, x_3, \ldots\ldots\ldots\ldots. x_n] = \frac{(-1)^n}{x_0.x_1.x_2,\ldots.x_n}$$

f. $f[x_0, x_1, x_2, x_3, \ldots\ldots\ldots\ldots. x_n] = \frac{1}{n!h^n}\Delta^n f_0$

g. $Prove\ \Delta^r(\alpha f(x) + \beta g(x) = \alpha\Delta^r f(x) + \beta\Delta^r g(x)\ ; r \ge 0$

h. For the following data, calculate the differences and calculate the forward difference polynomial. Interpolate at x= 0.25 & x= 0.35.

X	0.1	0.2	0.3	0.4	0.5
F(x)	1.4	1.56	1.76	2	2.28

i. Construct a Lagrangian Interpolating Polynomial for $f(x) = \ln(x)\ ;\quad x = 1,2,3$
Also estimate the value of L(1.5) & L(2.5). what is the error in approximation.

Section III

13. Obtain Piecewise Linear Interpolation.

X	0	1	2	3
y	0	1	4	3

Estimate the value of f(1.5) & f(2.5)

14. Obtain Piecewise Linear Interpolation.

X	1	2	4	8
y	3	7	21	73

Estimate the value of f(3) & f(7)

15. Derive the three point forward difference formula;

$$f'(x_1) = \frac{-f(x_{i+2}) + 4f(x_{i+1}) - 3f(x_i)}{x_{i+2} + x_i}\ with\ error\ O(h^2)$$

$Approximate\ f'(1), f'(2), and\ f'(3), from\ the\ following\ data.$

X	1	2	3	4	5
F(x)	2	4	8	16	32

16. Ordinary Differential Equation;

a. Apply **Euler's Method** to approximate the solution of the IVP;
$\frac{dy}{dx} = -yx^2$, y(1) = 2 over the interval [1,2] using five steps.

b. $\frac{dy}{dx} = -2xy^2$, y(0) = 1, Estimate y(0.4), using **Ralston Method**, with h = 0.2

c. $\frac{dy}{dx} = 1 - 2xy$, y(0) =0, Over the interval [0,1] , using 4th order **Runge-Kutta method**, with h = 0.5

d. $\frac{dy}{dx} = x + y$, y(0) =2, Over the interval [0,1] , using 4th order **Runge-Kutta method**, with h = 0.2

e. $\frac{dy}{dx} = 1 + \frac{y}{x}$, y(0) =1, find f(1.5) , using 4th order **Runge-Kutta method**, with h = 0.5

f. $\frac{dy}{dx} = y^2 + x^2$, y(0) =1, find y(1.5) , using 4th order **Modified Euler's method**, with h = 0.5

g. Apply **finite difference Method** to approximate the solution of the BVP;
$\frac{d^2y}{d^2x} = y + x, \quad 0 \le x \le 1$, y(0) = 2 , y(1) = 2.5 and h= 0.25.

h. Apply **finite difference Method** to approximate the solution of the BVP;
$\frac{d^2y}{d^2x} = y + x(x-4), \quad 0 \le x \le 4$, y(0) = 0 , y(1) =0 and n=4.

17. Numerical Integration;

a. Derive Trapezoidal rule with its error term. What is the degree of precision for this method?

b. Compute $\int_0^2 \frac{dx}{x}$, using Trapezoidal rule with n=8.

c. Compute $\int_1^2 \frac{dx}{1+x}$, using Simpson 1/3rd and Romberg integration rule with n=8.

d. Derive Composite Error formula for Trapezodal Method and Simpson Method.

e. Compute $\int_0^2 e^{-x^2} dx$, using Gaussian quadrature for n=2, 3, 4.

f. Compute $\int_0^1 \frac{dx}{1+x^2}$, using Simpson 1/3 rd rule.

g. Derive $f''(xi) = \frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1})}{h^2}$